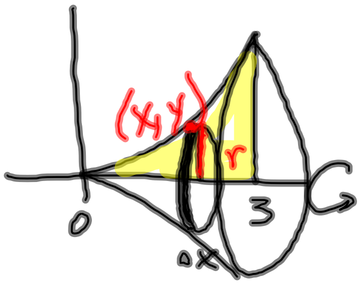
7.3 Volumes of Revolution

$y = x^2$

rotate the area around the x-axis



Δx = width of slice

volume of slice

$\Delta V_i = \pi r_i^2 \Delta x$

$V \approx \sum_{i=1}^{n} \pi r_i^2 \Delta x$ n slices

$V = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \pi r_i^2 \Delta x$

$r = y = x^2$

$$V = \int_0^3 \pi (x^2)^2 \, dx$$

$= 152.681$

$$\boxed{V = \int_a^b \pi r^2 \, dx}$$

r needs to be a function of x

disks

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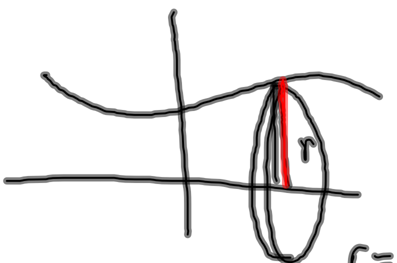
Region bounded by $y = 2 + x\cos x$ and the x-axis on $[-2, 2]$

rotate around the x-axis. Find the volume



$$\int_{-2}^{2} \pi \left(2 + x \cdot \cos x\right)^2 dx = 52.4288$$

$r = y = 2 + x\cos x$

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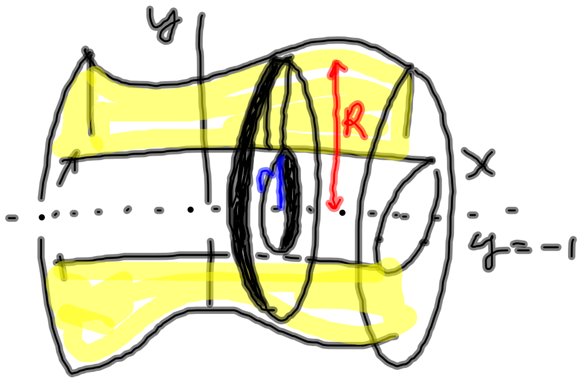
$y = 2 + x\cos x$, ~~x axis~~, $[-2, 2]$ about $y = -1$



method of washers

$$\boxed{\int_a^b \pi R^2 - \pi r^2 \, dx}$$

radius of hole

$R = 2 + x\cos x + 1$

$r = 1$

$$\int_{-2}^{2} \pi (2 + x\cos x + 1)^2 - \pi 1^2 \, dx$$

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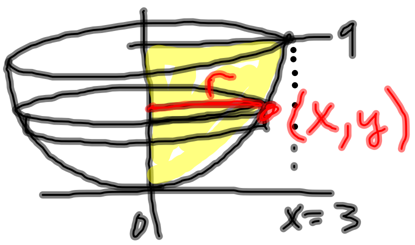
region bounded by $y = x^2$, $y = 9$, y axis



rotate about the y-axis

$$\boxed{\int_c^d \pi r^2 \, dy \quad \text{method of disks}}$$

$r = x = \sqrt{y}$

need r in terms of y

$$\int_0^9 \pi (\sqrt{y})^2 \, dy$$

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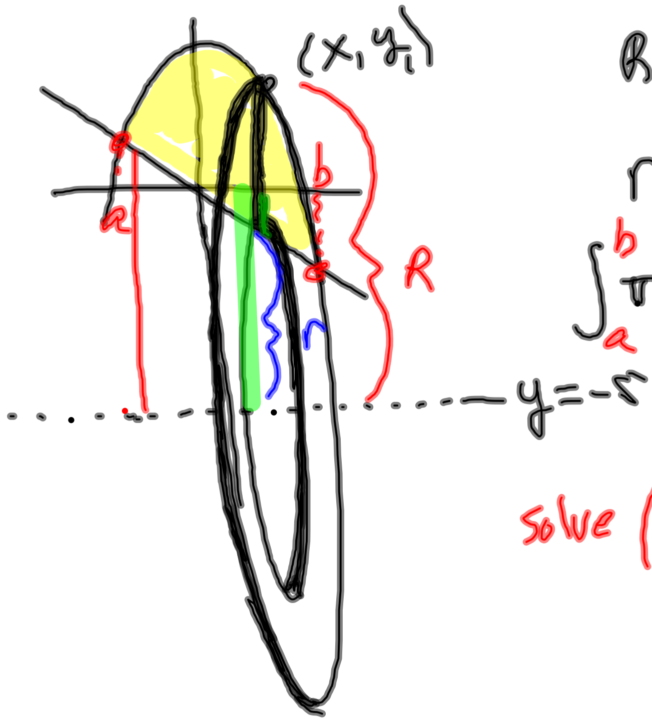
$y_1 = 4 - x^2, \quad y = -x \quad$ about $\quad y = -5$



$R = 4 - x^2 + 5$

$r = 5 + (-x)$

$$\int_a^b \pi (9 - x^2)^2 - \pi (5 - x)^2 \, dx$$

solve$(4 - x^2 = -x, x)$

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